Impact of Unit Failure on Forward Contracting

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Outline

• Introduction: unit availability
• Problem characterization
• Proposed solution
• Numerical simulations
• Future work
Power Producer

• Selling in the pool:
  – Risky because of the volatility of the prices
• Selling through forward contracts:
  – Hedging strategy against pool price volatility
  – Risky due to the probability of unit failure
Understanding the problem

- High pool prices volatility
- Risk-neutral producer

Generate energy ➔ Sell energy in the pool

Expected profit ↑
Standard deviation of profit ↑
Understanding the problem

- High pool prices volatility
- Risk-averse producer
- Unit failures neglected

Problem: If the unit fails, the producer has to buy energy in the pool even if the price is high
Understanding the problem

![Graph showing price and unit status over periods]

- **Price / Unit Status**
  - Unit on
  - Unit forced out
  - Pool price
Understanding the problem

- High pool prices volatility
- Risk-averse producer
- Unit failures considered

Generate energy ➔ Sell limited energy through forward contracts

- Expected profit ↓
- Standard deviation of profit ↓
Aim

Analyze the effect of the unit failure on forward contracting decisions for different values of risk level
Decision Framework

• Planning horizon: e.g., one year
• Two decisions:
  – Forward contracting at weekly / monthly /quarterly intervals
  – Pool trading throughout the planning horizon
Decision Framework

Forward contracting

One month

Pool trading
Forward Contracting

- Buy/sell a fixed MW power quantity (up to a maximum) during a certain future time period at a specified price

\[ \lambda_c^S = \lambda_c^B \]

![Diagram showing Forward Contract with axes for price ($/MWh) and power (MW). Arrow indicating 'Buy energy' pointing to the left and 'Sell energy' pointing to the right.](image)
Forward Contracting

• Advantages:
  – Hedging against pool price volatility
  – Lower standard deviation of profit

• Disadvantages:
  – Lower expected profit
  – Impossibility of selling energy through forward contracts if the unit is forced out
Uncertainty

- Uncertainty of pool price
- Uncertainty of unit availability
Modeling uncertainty

- **Price uncertainty:**
  - Scenario generation covering the study horizon
  - Scenario reduction
- **Unit availability:**
  - Scenario generation based on $MTTF$ (Mean Time To Failure) and $MTTR$ (Mean Time To Repair)
  - Scenario reduction
Characterizing price uncertainty

Pool price is treated as a stochastic variable:

Price scenario generation
Modeling unit availability

• Failure time modeled through an exponential distribution with mean $MTTF$
• Repair time modeled through an exponential distribution with mean $MTTR$

$$FOR\ (%)=\frac{MTTR}{MTTF+MTTR} \times 100$$
Modeling unit availability

• We simulate *Up Time* and *Down Time* as:

\[
Up \ Time = -MTTF \times \log(u_1) \\
Down \ Time = -MTTR \times \log(u_2)
\]

\(u_1\) and \(u_2\) are random variables uniformly distributed between 0 and 1
Modeling unit availability

Scenario j

Unit Status

Periods

$T_{F,1,j}$

$T_{F,2,j}$

$T_{F,3,j}$

$T_{R,1,j}$

$T_{R,2,j}$
Modeling unit availability

Status of the unit is treated as a stochastic variable:

Unit status scenario generation

![Graph showing unit status scenarios over periods](image-url)
Scenario reduction

- Needed to attain problem tractability
- Reduction technique based on the O.F. of a problem with one scenario (*Fast forward selection method*)
- Assessment via observation of the evolution of the objective function value with the # of scenarios
Model

- Two-stage stochastic programming approach:
  - Stochastic variables: pool price and unit availability
  - First stage variables: forward contracting decisions
  - Second stage variables: pool trading
Model

- Each scenario of the optimization problem contains:

\[
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & \ldots & 0 & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
24 & 37 & 41 & \ldots & 21 & 39 & 52
\end{bmatrix}
\]
Stochastic programming approach

Two-stage stochastic programming: Scenario tree

Forward contracting  Pool trading

Scenario 1
Scenario 2
Scenario n_Ω
Risk management

- Tradeoff between expected profit and risk due to profit variability
- Risk measure: CVaR (average profit in scenarios with lowest profit)
- CVaR advantage: linear formulation
Risk management

![Risk management diagram with CVaR and VaR](image.png)
Problem formulation

• Maximize \( \text{Expected profit} + \beta \times \text{CVaR} \)
• Subject to constraints associated with:
  – Energy balance
  – Forward contracts
  – CVaR

The tradeoff between expected profit and risk is enforced through the weighting factor \( \beta \in [0, \infty) \)
Expected profit

\[ E\{\text{profit}\} = E\{\text{revenue from pool involvement} - \text{generation cost}\} + \]
\[ + \text{forward contract purchases} - \text{forward contract cost} = \]

\[
= \sum_{\omega=1}^{N_W} \sum_{t=1}^{N_T} \pi_\omega \left( \lambda_{t\omega}^P E_t^P - E_t^G C \right) + \sum_{c=1}^{N_C} \lambda_c^S P_c^S \left( \sum_{t=T_c^0}^{T_c} L_t \right) - \sum_{c=1}^{N_C} \lambda_c^B P_c^B \left( \sum_{t=T_c^0}^{T_c} L_t \right)
\]
Energy balance

\[ E_{t\omega}^P + \sum_{c \in F_t} P_c^S L_t = \sum_{c \in F_t} P_c^B L_t + E_{t\omega}^G \quad \forall t, \forall \omega \]
Bounds

\[ E^G_{t \omega} \leq k_{tw} P^\max L_t \]
\[ E^G_{t \omega} \geq k_{tw} P^\min L_t \]

\[ P^S_c, P^B_c \geq 0 \]

\[ P^B_c \leq P^B_{c, \max} \]
\[ P^S_c \leq P^S_{c, \max} \]

\[ \forall t, \forall \omega \]
\[ \forall t, \forall \omega \]
\[ \forall c \]
\[ \forall c \]
\[ \forall c \]

Availability Parameter
Risk measure (CVaR)

CVaR is incorporated in the objective function through a weighting parameter $\beta$

$$CVaR = \xi - \frac{1}{1-\alpha} \sum_{\omega \in \Omega} \pi_\omega \eta_\omega$$

$$- \sum_{t=1}^{N_T} \lambda_{t_\omega} E_{t_\omega}^P - \sum_{c=1}^{N_C} \lambda_c^S P_c^S \left( \sum_{t=T_c^0}^{T_c} L_t \right) + \sum_{c=1}^{N_C} \lambda_c^B P_c^B \left( \sum_{t=T_c^0}^{T_c} L_t \right) +$$

$$+ \sum_{t=1}^{N_T} E_{t_\omega}^G C + \xi - \eta_\omega \leq 0 \quad \forall \omega$$

$$\eta_\omega \geq 0 \quad \forall \omega$$
Numerical simulations.
Stochastic Variable Characterization

• One month (28 days)
• Each day is divided into 6 periods of 4 hours each
• Total number of periods: \(28 \times 6 = 168\)
Numerical simulations. Prices scenarios

Original tree: 288 price scenarios → Final tree: 25 price scenarios
Numerical simulations.

Unit Status

• Simulation for three different cases:
  – Case 1: $FOR = 0\%$ (no failure)
  – Case 2: $FOR = 10\%$ ($MTTF=400$ $MTTR=40$)
  – Case 3: $FOR = 20\%$ ($MTTF=250$ $MTTR=60$)

• Original tree: 5000 availability scenarios → Final tree: 50 availability scenarios
Numerical simulations.
Reducing availability scenarios

FOR = 10%

FOR = 20%
Numerical simulations.
Forward contracts

- One month decision framework
- Five forward contracts:
  - Fixed price for each contract
  - Maximum power that can be sold/bought
  - 4 contracts with a duration of 1 week
  - 1 contract with a duration of the whole month
Numerical simulations.
Forward contracts

<table>
<thead>
<tr>
<th># contract</th>
<th>$P_c^{S,\text{Max}}$</th>
<th>$P_c^{B,\text{Max}}$</th>
<th>$\lambda_c^B$</th>
<th>$\lambda_c^S$</th>
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<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>52.88</td>
<td>52.88</td>
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<tr>
<td>5</td>
<td>50</td>
<td>50</td>
<td>52.57</td>
<td>52.57</td>
</tr>
</tbody>
</table>

Week 1 | Week 2 | Week 3 | Week 4
--- | --- | --- | ---
Contract 1 | | | |
Contract 2 | | | |
Contract 3 | | | Contract 4
Contract 5 | | | |
Results

\[ \beta \uparrow \implies \begin{cases} 
\text{expected profit} \downarrow \\
\text{profit standard deviation} \downarrow 
\end{cases} \quad FOR \uparrow \implies \text{expected profit} \downarrow \downarrow \]
Results

The higher the FOR, the lower the energy sold through FC.
Results

The producer sells less energy through forward contracts if the possibility of a unit failure increases.
Conclusion

• The energy sold through forward contracts by a unit decreases if its FOR increases
Simulation Data

- GAMS: cplex solver
- 2,6 GHz
- Linear optimization problem with continuous variables
- $4.2 \cdot 10^5$ variables
- $6.3 \cdot 10^5$ constraints
- Solution time: 3 minutes
Future work

- Forward contracts with increasing prices
- Include financial options
- Possibility of selling to end users directly ("gentailer")
- Analyze different scenario reduction techniques
Thanks for your attention!

GSEE: http://www.uclm.es/area/gsee/