Insuring Unit Failures in Electricity Markets

Salvador Pineda
Antonio Conejo
Miguel Carrión

Universidad de Castilla-La Mancha
(Spain)
2009
OUTLINE

- Introduction
- Model
- Case study
- Conclusions
OUTLINE

- Introduction
- Model
- Case study
- Conclusions
INTRODUCTION

Pool market
(price volatility)

Futures market
(fixed price)
INTRODUCTION

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INTRODUCTION

Pool market
(price volatility)

Futures market
(fixed price)

Production unit
### INTRODUCTION

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INTRODUCTION

Insurance

Pool price

Unit failure

Time
Swiss Re New Markets and Mirant Offer Generator Forced Outage Insurance Product.

Swiss Re's Electricity Price and Outage Protection (ELPRO) is a dual-trigger solution which protects against volume and market price by financially firming up generation whenever:
- Generating units suffer unplanned outages, and
- Electricity price exceeds a pre-agreed strike price.
INTRODUCTION

Sources of uncertainty
- Pool prices
- Unit availability

Forward contracts

Insurance policy
INTRODUCTION

Sources of uncertainty
- Pool prices
- Unit availability

Forward contracts

Insurance policy

Pool market

Futures market

Insurance?
INTRODUCTION

Sources of uncertainty
- Pool prices
- Unit availability

Forward contracts

Insurance policy

Scenario tree

Stochastic programming

Risk aversion

Pool market

Futures market

Insurance?
AIM

Analyze the effect of an insurance contract on the decisions of a power producer
Pool prices
Pool prices

Historical data

Pool price scenarios

ARIMA model
Sources of uncertainty
- Pool prices
- Unit availability

Forward contracts

Insurance policy

Scenario tree

Stochastic programming

Pool market

Futures market

Risk aversion

Insurance ?
Unit availability
Unit availability

Historical data

Failure time series = 20, 35, ....

~ exp(MTTF)

Repair time series = 12, 8, ....

~ exp(MTTR)

FOR(%) = \frac{MTTR}{MTTF + MTTF}

- t_F(1)
- t_F(2)
- t_R(1)
- t_R(2)

- Time
Unit availability

**Historical data**

- **OFF**
  - \( t_{F(1)} \)
  - \( t_{R(1)} \)

- **ON**
  - \( t_{F(2)} \)
  - \( t_{R(2)} \)

**Availability scenarios**

- \( t_{F} \sim \exp(MTTF) \)
- \( t_{R} \sim \exp(MTTR) \)
MODEL

Sources of uncertainty

Pool prices  Unit availability

Scenario tree

Pool price scenarios

Availability scenarios
Sources of uncertainty

Pool prices
Unit availability

Forward contracts

Insurance policy

Scenario tree

Stochastic programming

Risk aversion

Pool market
Futures market

Insurance?
Forward contracts
Forward contracts

- Specified quantity (MWh)
- Fixed price
- Future delivery period
Pool prices
Unit availability

Scenario tree

Stochastic programming

Forward contracts
Risk aversion

Insurance policy

Insurance ?

Pool market
Futures market
MODEL

Insurance policy
Insurance policy

- Initial premium
- Insured power
- Time period covered

Conditions:

- Pool price > Strike price
- Unit unavailable
Insurance policy

- Quantity paid by the insurance company to the producer
- Unit failure
- Strike price
- Pool price

Graph showing changes in pool price over time with specific periods highlighted.
Sources of uncertainty
- Pool prices
- Unit availability

Forward contracts

Insurance policy

Scenario tree

Stochastic programming

Risk aversion

Pool market

Futures market

Insurance ?
Stochastic programming

**Objective function**

Maximize $CVaR_\alpha(\text{profit}_\omega)$

**Constraints**

- Production unit limits
- Energy balance
- Forward characteristics
Stochastic programming

Objective function

Maximize \( CVaR_\alpha (\text{profit}_\omega) \)

Risk aversion

Constraints

Production unit limits
Energy balance
Forward characteristics

Probability

\( CVaR \quad \text{Var} \quad \text{Profit} \)

Probability

1 - \( \alpha \)
Stochastic programming

**Objective function**

Maximize $CVaR_\alpha(\text{profit}_\omega)$

$$CVaR_\alpha = \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_W} \pi_\omega \eta_\omega$$

$$- \text{profit}_\omega + \zeta - \eta_\omega \leq 0$$

$$\eta_\omega \geq 0$$

**Constraints**

- Production unit limits
- Energy balance
- Forward characteristics

Risk aversion
Stochastic programming

Objective function

Maximize $CVaR_\alpha(\text{profit}_\omega)$

$$\text{profit}_\omega = \sum_{t=1}^{N_T} \pi_\omega \left( \lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c=1}^{N_C} \lambda_c P_c T_c + s_I \left( -M_I + P_I \sum_{t \in G_\omega} (\lambda_{t\omega}^P - \lambda_I) T_t \right)$$

$\lambda_{t\omega}^P \to$ Pool price

$P_{t\omega}^P \to$ Power sold in the pool

$T_t \to$ Duration of time period

Constraints

Production unit limits

Energy balance

Forward characteristics
Stochastic programming

**Objective function**

Maximize $CVaR_\alpha(\text{profit}_\omega)$

$$\text{profit}_\omega = \sum_{t=1}^{N_T} \pi_\omega \left( \lambda_{t\omega} P_{t\omega} T_t - C(P^G_{t\omega}) \right) + \sum_{c=1}^{N_C} \lambda_c P_c T_c + s_I \left( -M_I + P_I \sum_{t \in G_\omega} (\lambda^P_{t\omega} - \lambda_I) T_t \right)$$

$C(\cdot) \rightarrow$ Cost function

$P^G_{t\omega} \rightarrow$ Generated power

**Constraints**

- Production unit limits
- Energy balance
- Forward characteristics
**Stochastic programming**

**Objective function**

Maximize $CVaR_\alpha(profit_\omega)$

$$profit_\omega = \sum_{t=1}^{N_T} \pi_\omega \left( \lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c=1}^{N_C} \lambda_c P_c T_c + s_I \left( -M_I + P_I \sum_{t \in G_\omega} (\lambda_{t\omega}^P - \lambda_I) T_t \right)$$

- $\lambda_c \rightarrow$ Forward price
- $P_c \rightarrow$ Power sold through forward contract
- $T_c \rightarrow$ Forward contract duration

**Constraints**

- Production unit limits
- Energy balance
- Forward characteristics
Stochastic programming

**Objective function**

Maximize \( CVaR_\alpha(\text{profit}_\omega) \)

\[
\text{profit}_\omega = \sum_{t=1}^{N_T} \pi_\omega \left( \lambda_{t\omega} P_{t\omega} T_t - C(P^G_{t\omega}) \right) + \sum_{c=1}^{N_C} \lambda_c P_c T_c + s_I \left( -M_I + P_I \sum_{t \in G_\omega} (\lambda_{t\omega}^P - \lambda_I) T_t \right)
\]

- \( s_I \) → Binary variable
- \( M_I \) → Initial premium
- \( P_I \) → Insured power
- \( t \in G_\omega \) ⟷ \( k_{t\omega} = 0 \) and \( \lambda_{t\omega}^P \geq \lambda_I \)
- \( \lambda_I \) → Strike price

**Constraints**

- Production unit limits
- Energy balance
- Forward characteristics
Stochastic programming

**Objective function**
Maximize \( CVaR_\alpha(\text{profit}_\omega) \)

**Constraints**
Production unit limits

\[
\begin{align*}
P_{\max} & \geq u_{t\omega} k_{t\omega} P_{t\omega}^G \\
& \geq P_{\min}
\end{align*}
\]

Input data (availability scenarios)
Binary variable (on/off)

**Balance energy**
**Forward characteristics**
Stochastic programming

Objective function
Maximize $CVaR_\alpha(profit_\omega)$

Constraints
Production unit limits
Energy balance

Forward characteristics

$$P_{t\omega}^G = \sum_{c \in F_t} P_c + P_{t\omega}^P$$

$$P_{t\omega}^P \geq (k_{tw} - 1) \sum_{c \in F_t} P_c$$
Stochastic programming

**Objective function**

Maximize $CVaR_{\alpha}(\text{profit}_\omega)$

**Constraints**

- Production unit limits
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- Forward characteristics

$$P_c \leq P_c^{\text{max}}$$
MODEL

Sources of uncertainty
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- Unit availability

Forward contracts

Insurance policy

Risk aversion

Stochastic programming

Scenario tree

Pool market

Futures market

Insurance?
MODEL

Pool market

Futures market

Insurance?
CASE STUDY

- 3 months
- 4 forward contracts (3 monthly and 1 quarterly)
- 1 Insurance contract
  - Premium: 100,000 €
  - Insured power: 75 MW
  - Strike price: 10 €/MWh
- 300 pool price scenarios reduced to 50
- 10,000 availability scenarios reduced to 200
- Generating unit
  - $P_{\text{max}} = 500$ MW & $P_{\text{min}} = 50$ MW
  - Three FOR values: 0, 4 and 8%
  - Piecewise lineal cost function
## CASE STUDY

### Without insurance

<table>
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<tr>
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Critical premium: the maximum amount that a producer is willing to pay in exchange for a given insurance contract.
Critical premium.

CVaR is a coherent risk measure: $\text{CVaR}_\alpha (Y + c) = \text{CVaR}_\alpha (Y) + c$
CASE STUDY

- Critical premium.

Case 1 ($s_1 = 1$):
\[ \text{CVaR}_1^P = \text{CVaR}_0^P - M_I \]

\[ \text{CVaR}_\alpha(Y + c) = \text{CVaR}_\alpha(Y) + c \]
Critical premium.

\[ \text{CVaR}_\alpha (Y + c) = \text{CVaR}_\alpha (Y) + c \]

Case 1 \((s_1 = 1)\):
\[ \text{CVaR}_1^P = \text{CVaR}_0^P - M_1 \]

Case 2 \((s_1 = 0)\):
\[ \text{CVaR}_2^P = \text{CVaR}_\infty^P \]

\( M_1 \text{ (euro)} \)
CASE STUDY

- Critical premium.

\[
CVaR_\alpha(Y + c) = CVaR_\alpha(Y) + c
\]

Case 1 \((s_I = 1)\):
\[
CVaR_1^P = CVaR_0^P - M_I
\]

Case 2 \((s_I = 0)\):
\[
CVaR_2^P = CVaR_\infty^P
\]

\[
CVaR_1^P = CVaR_2^P
\]
\[
CVaR_0^P - M_I^{P*} = CVaR_\infty^P
\]
\[
M_I^{P*} = CVaR_0^P - CVaR_\infty^P
\]
CASE STUDY

\[ P_1 = 75 \text{ MW} \]

\[ \lambda_1 \text{ (euro/MWh)} \]

\[ M_{1}^{P*} \text{ (euro)} \]

- FOR = 4% $\alpha^P = 0$
- FOR = 4% $\alpha^P = 0.8$
- FOR = 4% $\alpha^P = 0.95$
- FOR = 8% $\alpha^P = 0$
- FOR = 8% $\alpha^P = 0.8$
- FOR = 8% $\alpha^P = 0.95$

\[ 7 \times 10^5 \]
## CASE STUDY

**Producer**

<table>
<thead>
<tr>
<th>( \alpha^P )</th>
<th>( M_i^{P*}(€) )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>173.275</td>
</tr>
<tr>
<td>0.3</td>
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</tr>
<tr>
<td>0.5</td>
<td>214.615</td>
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<td>0.9</td>
<td>358.210</td>
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**Insurance company**

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<tbody>
<tr>
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<tr>
<td>0.3</td>
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\[
P_i = 75\text{MW} \quad \lambda_i = 10€ / \text{MWh} \quad \text{FOR} = 8\%
\]
## CASE STUDY

### Producer vs. Insurance Company

<table>
<thead>
<tr>
<th>$\alpha^P$</th>
<th>$M^P_*(€)$</th>
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\[ P_I = 75 \text{MW} \quad \lambda_I = 10€ / \text{MWh} \quad \text{FOR} = 8\% \]
### CASE STUDY

**Producer**

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</tr>
<tr>
<td>0.5</td>
<td>214.615</td>
</tr>
<tr>
<td>0.9</td>
<td>358.210</td>
</tr>
</tbody>
</table>

**Insurance company**

<table>
<thead>
<tr>
<th>$\alpha^S$</th>
<th>$M_{IS^*}(€)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>173.275</td>
</tr>
<tr>
<td>0.3</td>
<td>242.071</td>
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<tr>
<td>0.5</td>
<td>302.974</td>
</tr>
<tr>
<td>0.9</td>
<td>563.779</td>
</tr>
</tbody>
</table>

$$P_I = 75 M W \quad \lambda_I = 10€ / M W h \quad \text{FOR} = 8\%$$
OUTLINE

- Introduction
- Model
- Case study
- Conclusions
CONCLUSIONS

Stochastic programming
CONCLUSIONS

Insurance → Futures market
CONCLUSIONS

Insurance  \rightarrow  Futures market
FOR  \rightarrow  Critical premium
Risk aversion  \rightarrow  Critical premium
CONCLUSIONS

Producer → Risk → Insurance → Insurance company
CONCLUSIONS

Thank you!

Questions?

www.uclm.es/area/gsee