Options to Hedge Against Producer Risks in Electricity Markets

Salvador Pineda
Antonio Conejo
November 2010
Outline

- Introduction
- Model
- Case study
- Conclusions
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- Introduction
- Model
- Case study
- Conclusions
Introduction

- How do electricity options work?
- How can electricity options be modeled?
- How do electricity options reduce price risk?
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?
Introduction

Pool market
(price risk)
Introduction

Pool market
(price risk)

Futures market
(fixed price)
Introduction

Pool market
(price risk)

Production unit
(availability risk)
Introduction

Pool price

Time

Unit failure

Pool price
Introduction

**Pool market** (price risk)

**Production unit** (availability risk)

**Futures market** (fixed price)
Introduction

Forward contract

- Fixed price
- Obligation to buy/sell
- No cost

Option contract

- Fixed price
- Right to buy/sell
- Option price
Option contract

- Two positions (buyer and seller of the option)
- Put options (right to sell)
- Call options (right to buy)
- European options (exercised at expiration)
- American options (exercised any time until expiration)
Introduction

- Two positions (buyer and seller of the option)
- **Put options** (right to sell)
- Call options (right to buy)
- **European options** (exercised at expiration)
- American options (exercised any time until expiration)
## Option contract

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<thead>
<tr>
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<th>No. of Contr.</th>
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Introduction

Basic idea

Period 1

- Sign a forward contract to sell electricity during period 2

Period 2

- Obligation to sell the agreed electricity at the agreed price
Introduction

profit

loss

\lambda_{(c)}

\lambda^P

Forward
Introduction

Basic idea

Sign a put option to sell electricity during period 2
Introduction

Basic idea

Sign a put option to sell electricity during period 2

Option is exercised to hedge against low prices
Introduction

Basic idea

Sign a put option to sell electricity during period 2

Option is not exercised to obtain high profits
Introduction
Introduction

\[ \lambda^S_{(o)} = \lambda_{(c)} \]

\[ \lambda^S_{(o)} + \lambda^O_{(o)} \]

Limited losses!
Introduction

Basic idea

Sign a put option to sell electricity during period 2

Option is not exercised to hedge against unit failures
Introduction

Basic idea

Sign a put option to sell electricity during period 2

Option is exercised
Introduction

- How do electricity options work?
- How can electricity options be modeled?
- How do electricity options reduce price risk?
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?
Introduction

OBSERVE

DECIDE

DECIDE

Multi-stage stochastic programming
Introduction

Period 1

- Option purchase
- Forward contracting

Period 2

- Option exercise
Introduction

Scenario tree
Introduction

- How do electricity options work? ✓
- How can electricity options be modeled? ✓
- How do electricity options reduce price risk?
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?
Aim

Analyze electricity options to manage the two main risks faced by power producers: price and availability risks.
Outline

- Introduction
- Model
- Case study
- Conclusions
Outline

- Introduction
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- Case study
- Conclusions
Model

Sources of uncertainty
- Pool prices
- Unit availability

Forward contracts
Option contracts
Model

Sources of uncertainty
Pool prices
Unit availability
Forward contracts
Option contracts

Pool market
Options
Futures market
Model

Sources of uncertainty

Pool prices
Unit availability

Scenario tree

Forward contracts

Option contracts

Risk aversion

Pool market

Futures market

Options

Multi-stage stochastic programming

Model
Model

Pool prices

Historical data

Pool price scenarios

Time series model
Unit availability

**Historical data**

Failure time series $= 20, 35, \ldots$  
$\sim \exp(\text{MTTF})$

Repair time series $= 12, 8, \ldots$  
$\sim \exp(\text{MTTR})$

FOR($\%$) $= \frac{\text{MTTR}}{\text{MTTR} + \text{MTTF}}$
Model

Unit availability

\[
\begin{align*}
t_F & \sim \exp(\text{MTTF}) \\
t_R & \sim \exp(\text{MTTR})
\end{align*}
\]

\[
p(u_t = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu(u_0 - 1) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}
\]

\[
\lambda = \frac{1}{\text{MTTF}} \quad \mu = \frac{1}{\text{MTTR}}
\]
Model

Unit availability

\[
\begin{align*}
\begin{cases}
  t_F &\sim \exp(\text{MTTF}) \\
  t_R &\sim \exp(\text{MTTR})
\end{cases}
\end{align*}
\]

\[
p(u_t = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu(u_0 - 1) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}
\]

MTTF<<T

MTTF>>T

Graphs showing probability vs. time for different conditions.
Unit availability

Historical data

Availability scenarios

\[ t_F \sim \exp(\text{MTTF}) \]
\[ t_R \sim \exp(\text{MTTR}) \]
Model

Sources of uncertainty

Pool prices

Unit availability

Scenario tree

Pool price scenarios

Availability scenarios
Model

Forward contracts

- Specified quantity (MW)
- Fixed price
- Future delivery period
Option contract

- Specified quantity (physical delivery)
- Strike price
- Option price
- Time period covered
- Time to decide whether it is exercised
Model

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_\omega)$

Constraints

- Production unit bounds
- Energy balances
- Forward and option constraints
- Nonanticipativity constraints
Model

Stochastic programming

Objective function

Maximize \( CVaR_\alpha(\text{profit}_\omega) \)

\[
CVaR_\alpha = \zeta - \frac{1}{1 - \alpha} \sum_{\omega=1}^{N_W} \pi_\omega \eta_\omega \\
- \text{profit}_\omega + \zeta - \eta_\omega \leq 0 \\
\eta_\omega \geq 0
\]

Constraints

- Production unit bounds
- Energy balances
- Forward and option constraints
- Nonanticipativity constraints

Risk aversion

![Graph showing CVaR and VaR](image)
Second-stage decisions are made knowing the scenario realization during period 1 but still facing uncertainty related to period 2.
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Case study

- 2 months (P1 = first month, P2 = second month)
- Generating unit
  - Pmax = 350 MW, Pmin = 50 MW, C = 12 €/MWh (linear)
  - Three FOR values: 0, 5 and 10%
- 30 pool-price scenarios (ARIMA)
- 30 availability scenarios for each value of FOR
- 2 forward contracts, one for each month
- 1 put option spanning the second month
Case study

- (a) sell 350MW during the second month at 21€/MWh. No re-trading in stage 2.
- (b) buy a put option to sell 350MW during the second month at 21€/MWh. Option price = 0.1€/MWh
## Case study

<table>
<thead>
<tr>
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- (a) sell 350MW during the second month at 21€/MWh. No re-trading in stage 2.
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**OPTION > FORWARD**
Case study

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Period 1                              Period 2

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### OPTIONS reduce price risk

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Case study

- How do electricity options work? √
- How can electricity options be modeled? √
- How do electricity options reduce price risk? √
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?
## Case study

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↑ risk aversion → **OPTION ≈ FORWARD**
## Case study

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### Period 1

- Stage 1
- Stage 2

### Period 2

- Stage 1
- Stage 2
- Stage 3

### Table:

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# Case study

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### Diagram:

**Period 1**

1. Stage 1
2. Stage 2
3. Stage 3

**Period 2**

- **E_{2}(\lambda^p)**
  - 26.01
  - 20.97
  - 20.28

<table>
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### Diagram

- **Period 1**
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- **Stage 1**
- **Stage 2**
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## Case study

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### Diagram

- **Stage 1**
- **Stage 2**
- **Stage 3**

### Table: $E_2(\Lambda^p)$

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<tr>
<th>$k_{NT1}$</th>
<th>$E_2(\Lambda^p)$</th>
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## Case study

### OPTIONS reduce availability risk

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</tr>
</tbody>
</table>
Case study

- How do electricity options work? ✓
- How can electricity options be modeled? ✓
- How do electricity options reduce price risk? ✓
- How do electricity options reduce availability risk? ✓
- When is an option contract more profitable than a forward contract?
Case study

- MAXIMUM OPTION PRICE that the power producer is willing to pay for a given option contract.
Case study

MAXIMUM OPTION PRICE

α = 0

α = 0.5

Case study

MAXIMUM OPTION PRICE

α = 0

α = 0.5

Case study

MAXIMUM OPTION PRICE

α = 0

α = 0.5
Case study

MAXIMUM OPTION PRICE

\( \alpha = 0 \)

\( \alpha = 0.5 \)

**Strike price >> Spot**

OPTION \(\times\) FORWARD
Case study

MAXIMUM OPTION PRICE

\[ \alpha = 0 \]

\[ \alpha = 0.5 \]

Strike price \ll \text{Spot} \quad \text{OPTION} \quad \text{FORWARD}
Case study

MAXIMUM OPTION PRICE

$\alpha = 0$

$\alpha = 0.5$

↑ risk aversion $\rightarrow$ OPTION $\approx$ FORWARD
Case study

- MAXIMUM OPTION PRICE

$$\alpha = 0$$

FOR not relevant

$$\alpha = 0.5$$

↑FOR → OPTION
\[ t_F \sim \exp(\text{MTTF}) \]
\[ t_R \sim \exp(\text{MTTR}) \]

\[ p(u_t = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu(u_0 - 1) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \]

Unit availability

MTTF $<< T$

MTTF $>> T$
Case study

MAXIMUM OPTION PRICE (impact of MTTF)

FOR = 5%

FOR = 10%
Outline

- Introduction
- Model
- Case study
- Conclusions
Outline

- Introduction
- Model
- Case study
- Conclusions
Conclusions

Multi-stage stochastic programming
Conclusions

Options → Price and availability risk
Conclusions

Options → Price and availability risk → Forwards

Options
Conclusions

- Strike price ≈ Spot
- FOR with ↑MTTF
- Option price < Max. Option
- Price and availability risk

Options ↔ Options ↔ Forwards
Conclusions

- Strike price $\approx$ Spot
- FOR with $\uparrow$MTTF
- Option price $<$ Max. Option

Price and availability risk
Conclusions

- Forward price > Spot
- ↓FOR
- ↓MTTF

Price and availability risk
Conclusions

- Options
- Forwards

Price and availability risk

- Forward price > Spot
- ↓FOR
- ↓MTTF
Conclusions

- How do electricity options work? ✓
- How can electricity options be modeled? ✓
- How do electricity options reduce price risk? ✓
- How do electricity options reduce availability risk? ✓
- When is an option contract more profitable than a forward contract? ✓
Thank you!

Questions?

www.uclm.es/area/gsee
Model

Stochastic programming

Objective function

Maximize \( CVaR_\alpha(\text{profit}_\omega) \)

\[
\text{profit}_\omega = \sum_{t=1}^{N_T} \left( \lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c_1=1}^{N_{c_1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{c_2}} \left( \lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2 \right) T_{c_2} + \sum_{o=1}^{N_O} y_o \left( -\lambda_o^O P_o + y_{o\omega} \lambda_o^S P_o \right) T_o
\]

Constraints

- Production unit bounds
- Energy balances
- Forward and option constraints
- Nonanticipativity constraints
Model

Stochastic programming

**Objective function**

Maximize \( CVaR_\alpha(\text{profit}_\omega) \)

\[
\text{profit}_\omega = \sum_{t=1}^{N_T} \left( \lambda_{t_\omega}^P P_{t_\omega}^P T_t - C(P_{t_\omega}^G) \right) + \sum_{c_1=1}^{N_{C_1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C_2}} \left( \lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2}^2 P_{c_2}^2 \right) T_{c_2} + \sum_{o=1}^{N_O} \nu_o \left( -\lambda_o^O P_o + y_{o_\omega} \lambda_o^S P_o \right) T_o
\]

\( C(\cdot) \rightarrow \text{Cost function} \)

\( P_{t_\omega}^G \rightarrow \text{Generated power} \)

**Constraints**

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints
Stochastic programming

**Objective function**

Maximize \( CVaR_\alpha (\text{profit}_\omega) \)

**Pool**

\[
\text{profit}_\omega = \sum_{t=1}^{N_T} (\lambda_{t_\omega}^P P_{t_\omega}^T - C(P_{t_\omega}^G))
\]

**Cost**

\[
\sum_{c_1=1}^{N_{C_1}} \lambda_{c_1} P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C_2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2}^2 P_{c_2}^2)T_{c_2} + \sum_{o=1}^{N_{O}} v_o (-\lambda_{o}^P P_o + y_{o_\omega} \lambda_{o}^S P_o)T_o
\]

**Forward 1**

Contracts spanning period 1

\( \lambda_{c_1}^1 \rightarrow \text{Forward price} \)

\( P_{c_1}^1 \rightarrow \text{Sold power} \)

\( T_{c_1} \rightarrow \text{Forward contract duration} \)

**Constraints**

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints
Stochastic programming

Objective function
Maximize $CVaR_\alpha(\text{profit}_\omega)$

$$\text{profit}_\omega = \sum_{t=1}^{N_T} (\lambda_{t\omega}^P P_{t\omega}^P T_t) - C(P_{t\omega}^G) + \sum_{c_1=1}^{N_{C_1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C_2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2) T_{c_2} + \sum_{o=1}^{N_O} v_o (-\lambda_o^O P_o + y_{o\omega}^S P_{o}) T_o$$

Constraints
Production unit bounds
Energy balances
Forward and option constraints
Nonanticipativity constraints

Contracts spanning period 2
$$\lambda_{c_2}^1, \lambda_{c_2\omega}^2 \rightarrow \text{Forward price in stage } 1/2$$
$$P_{c_2}^1, P_{c_2\omega}^2 \rightarrow \text{Sold power in stage } 1/2$$
$$T_{c_2} \rightarrow \text{Forward contract duration}$$
Stochastic programming

Objective function

Maximize $\text{CVaR}_\alpha(\text{profit}_\omega)$

$\text{profit}_\omega = \sum_{t=1}^{N_T} \left( \lambda^P_{t\omega} P^P_{t\omega} T_t - C(P^G_{t\omega}) \right) + \sum_{c_1=1}^{N_{C_1}} \lambda^1_{c_1} P^1_{c_1} T_{c_1} + \sum_{c_2=1}^{N_{C_2}} \left( \lambda^1_{c_2} P^1_{c_2} + \lambda^2_{c_2\omega} P^2_{c_2\omega} \right) T_{c_2} + \sum_{o=1}^{N_O} v_o (-\lambda^O_o P_o + y_{o\omega} \lambda^S_o P_o) T_o$

Constraints

Production unit bounds
Energy balances
Forward and option constraints
Nonanticipativity constraints

Option

$v_o \rightarrow$ Option purchase (1/0)
$\lambda^O_o, \lambda^S_o \rightarrow$ Option and strike price
$y_{o\omega} \rightarrow$ Option exercise (1/0)
$P_o \rightarrow$ Sold power
$T_o \rightarrow$ Option duration
Model

Stochastic programming

**Objective function**

Maximize \( CVaR_\alpha(\text{profit}_\omega) \)

**Constraints**

Production unit bounds

\[ u_{t_\omega} k_{t_\omega} P_{\text{max}} \geq P_{t_\omega} \geq u_{t_\omega} k_{t_\omega} P_{\text{min}} \]

- Constant (availability scenario)
- Binary variable (on/off)

**Energy balances**

**Forward and option constraints**

**Nonanticipativity constraints**
Model

Stochastic programming

Objective function
Maximize $CVaR_\alpha(\text{profit}_\omega)$

Constraints
Production unit bounds
Energy balances

$$P_{t\omega}^G = \sum_{c_1 \in F_t^1} P_{c_1}^1 + \sum_{c_2 \in F_t^2} (P_{c_2}^1 + P_{c_2\omega}^2) + \sum_{o \in O_t} v_o y_{o\omega} P_o + P_{t\omega}^P$$

Forward and option constraints
Nonanticipativity constraints
Model

Stochastic programming

**Objective function**
Maximize $CVaR_\alpha(\text{profit}_\omega)$

**Constraints**
Production unit bounds
Energy balances
Forward and option constraints

$$P_{c_1}^1 \geq 0$$

$$P_{c_2}^1 + P_{c_2 \omega}^2 \geq 0$$

$$P_o \geq 0$$

Nonanticipativity constraints
Case study

- (a) sell 350MW during the second month at 21€/MWh. Retrading in stage 2 at the following prices:

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<th>$\lambda^2_{c2\omega}$</th>
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- (b) buy a put option to sell 350MW during the second month at 21€/MWh. Option price = 0.1€/MWh

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