Scenario Reduction for Futures Market Trading in Electricity Markets

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Scenario reduction
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Stochastic programming in electricity markets

• Uncertainty is present in electricity market problems

• Solution: Use of stochastic programming models
  
  • Cabero et al., (2005)
  • Plazas et al., (2005)
  • Sen et al., (2006)
  • Li et al., (2007)
  • Wu et al., (2008)
  • Conejo et al., (2008)
  • …
Examples

• Producer: to determine the sales in the futures market and the pool

• Retailer: to determine the purchases in the futures markets and the selling price offered to its potential clients
Scenario set

- The uncertain parameters (stochastic processes) are represented by finite sets of plausible scenarios.
- For example: pool price.
Tractability

• The required number of scenarios for representing a stochastic process is very large (e.g., thousands of scenarios)

• This leads to stochastic programming problems with millions of constraints and variables → untractable problems

• Solution:
  – Decomposition techniques
  – Scenario reduction procedures
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Objective

- To reduce the original number of scenarios while still retaining the essential features of the original scenario set

- The solution obtained using the reduced scenario tree should be “close” to that obtained with the original tree
In two-stage stochastic linear programming problems, it is possible to measure the ‘similarity’ between two set of scenarios \((\Omega, \Omega_S)\) by using a probability distance (e.g., Kantorovich distance):

\[
D_K(\Omega, \Omega_S) = \min_{\eta} \left\{ \sum_{\omega \in \Omega} \sum_{\omega' \in \Omega_S} c(\omega, \omega') \eta(\omega, \omega') : \right. \\
\left. \eta(\omega, \omega') \geq 0, \forall \omega \in \Omega, \forall \omega' \in \Omega_S, \right. \\
\left. \sum_{\omega' \in \Omega_S} \eta(\omega, \omega') = \pi_\omega, \forall \omega \in \Omega, \right. \\
\left. \sum_{\omega \in \Omega} \eta(\omega, \omega') = \tau_{\omega'}, \forall \omega' \in \Omega_S \right\}
\]
Probability distance

$c(\omega, \omega')$ is named **cost function** and is

- Nonnegative, continuous, and symmetric
- $c(\omega, \omega')$ is given by a norm
Probability distance

- If the random variable $\lambda$ is represented by the sets of scenarios $\Omega$ and $\Omega_S$, where $\Omega_S$ is a subset of $\Omega$, the Kantorovich distance (1) becomes (Dupačová et. al, 2003):

$$D_K(\Omega, \Omega_S) = \sum_{\omega \in \Omega \setminus \Omega_S} \pi_\omega \min_{\omega' \in \Omega_S} c(\omega, \omega')$$

(2)

where

$$c(\omega, \omega') = \|\lambda_\omega - \lambda_{\omega'}\|$$
Probability distance

• Expression (2) allows computing directly the Kantorovich distance without solving any mathematical problem.

• This has motivated several iterative scenario reduction algorithms:
  – For example, the forward selection algorithm (Heitsch and Römisch, 2003)
Forward selection algorithm

- Iterative greedy procedure starting with the empty tree
- At each iteration, the scenario which minimizes the Kantorovich distance between $\Omega$ and $\Omega_S$ is selected
- Based on qualitative and quantitative stability results
Forward selection algorithm

1. Compute the cost function: \[ c(\omega, \omega') = \| \lambda_\omega - \lambda_{\omega'} \|, \ \forall \omega, \omega' \in \Omega \]

2. Initialize the reduced set of scenarios: \( \Omega_S \leftarrow \emptyset \)

3. Select the scenario that included in the reduced set minimizes the Kantarovich distance between \( \Omega \) and \( \Omega_S \)

4. Repeat until a sufficient number of scenarios is selected

5. Update the probability associated with the selected scenarios
New reduction technique

• Based on the fast forward algorithm we propose a novel scenario reduction technique

• The new technique is a heuristic procedure to be used in risk-neutral two-stage stochastic programming problems within an electricity markets framework
New reduction technique

• Previous techniques are based on the difference between stochastic processes in different scenarios

\[ c(\omega, \omega') = \| \lambda_\omega - \lambda_{\omega'} \| \]

• The new technique is based on the difference between objective functions associated with different scenarios

\[ c(\omega, \omega') = \| z_\omega - z_{\omega'} \| \]
New reduction technique

where:

\( z_\omega \) is the objective function associated with the original stochastic programming problem in which:

- The value of the stochastic process \( \lambda \) is replaced by its realization in scenario \( \omega \), \( \lambda_\omega \)

- The first-stage variables are fixed to those obtained by solving the deterministic problem

\( z_\omega \) is obtained solving a single-scenario problem!!!
Algorithm

1) Solve the deterministic problem associated with the stochastic problem to fix first-stage decisions.

2) For each scenario $\omega$ solve a single-scenario problem to obtain $z_{\omega}$.

3) Compute the cost function, $c(\omega, \omega') = |z_\omega - z_{\omega'}|$, for each pair of scenarios $\omega$ and $\omega'$.

4) Select the scenario that included in the reduced set of scenarios minimizes the Kantorovich distance between the reduced and the original sets.

5) Repeat 4) until a sufficient number of scenarios is selected.
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Case 1. Power producer

- Futures market involvement of a power producer (Conejo et al., 2008)

- Two-stage stochastic programming problem:
  - Maximize the expected profit subject to:
    - Forward contracting constraints
    - Operating constraints
    - Energy balance constraints

- Stochastic process: pool price
Pool price scenarios for the producer problem

- 200 scenarios (heuristic scenario generation procedure)
Expected profit vs. number of scenarios

![Graph showing expected profit vs. number of scenarios with Traditional and New algorithms.](image.png)
Expected profit vs. number of scenarios

- Traditional algorithm
- New algorithm

Expected profit (€ million) vs. number of pool price scenarios.
Profit standard deviation vs. number of scenarios

![Graph showing profit standard deviation vs. number of pool price scenarios. The graph compares Traditional algorithm and New algorithm. The x-axis represents the number of pool price scenarios, ranging from 0 to 200. The y-axis represents the standard deviation in € million, ranging from 0 to 4.]
Profit standard deviation vs. number of scenarios

![Graph showing the comparison between standard deviation and the number of pool price scenarios for Traditional and New algorithms. The graph demonstrates that the New algorithm consistently has a lower standard deviation across various numbers of scenarios.](image-url)
Distance between the original and reduced trees

**Traditional algorithm**

- Price tree distance (normalized)
- Number of pool price scenarios

**New algorithm**

- Profit tree distance (normalized)
- Number of pool price scenarios

Scenario reduction
Case 2. Power retailer

- Forward contracting and price determination (Carrión et al., 2007)

- Two-stage stochastic programming problem:
  - Maximize the expected profit subject to:
    - Forward contracting constraints
    - Client constraints
    - Energy balance constraints
    - Revenue constraints

- Stochastic processes: pool price and client demand
Pool price scenarios for the retailer problem

- 200 scenarios (ARIMA model)
Expected profit vs. number of scenarios

![Graph showing expected profit vs. number of scenarios for traditional and new algorithms.](image-url)
Expected profit vs. number of scenarios

![Graph showing expected profit vs. number of scenarios.

- Traditional algorithm (blue)
- New algorithm (red)

The graph plots the expected profit in euros million against the number of pool price scenarios. The expected profit tends to stabilize as the number of scenarios increases.

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Profit standard deviation vs. number of scenarios

![Graph showing profit standard deviation vs. number of pool price scenarios. The graph compares traditional and new algorithms.](image-url)
Profit standard deviation vs. number of scenarios

- Traditional algorithm
- New algorithm

Number of pool price scenarios vs. standard deviation of the profit (€ million)
Distance between the original and reduced trees

![Graph showing distance between original and reduced trees using two algorithms: Traditional and New. The x-axis represents the number of pool price scenarios, while the y-axis shows the price tree distance and profit tree distance (normalized). The Traditional algorithm graph shows a decreasing trend, while the New algorithm graph also shows a decreasing trend but with a different curve.](image-url)
Computational burden

• Power producer problem with 1000 scenarios

• Reduction 1000 → 100 scenarios (New Algorithm): 284 s
  – 247 s: to solve 1000 single-scenario problems
  – 10 s: to compute the cost function
  – 11 s: to carry out the fast-forward selection
  – 16 s: to solve the producer problem with 100 scenarios

• Solution with 1000 scenarios: 467 s (↑ 64 %)
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Conclusions and Future Research

- The proposed technique is able to select a reduced number of scenarios that represents the original set.

- The proposed technique has a higher computational burden than previously reported techniques.

- Qualitative and quantitative stability results are needed to assess the quality of the method (future research).

- Extension of the technique to handle stochastic programming problems with risk measures.
Thanks for your attention!

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